FRACTALS AND HEAVY TAILS IN THE ROMANIAN STOCK MARKET

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Abstract. *The Fractal Market Hypothesis and the existence of fat tails are investigated for the BET Index of Bucharest Stock Exchange. In addition to this, an uncertainty index is defined in order to explain large deviations in stock market returns.*

Keywords: fractals, heavy tails, uncertainty.

1. INTRODUCTION

This Fractal Market Hypothesis (FMH) was first introduced by Peters (1989) as a response to the Efficient Market Hypothesis (EMH).

The Fractal Market Hypothesis is based on four essential elements describing the stock market:

- The market is stable and liquid enough when investors have different time horizons;
- Investors maintain the time horizon of the investments independently of informational changes;
- Available information are not automatically reflected by prices;
- The evolution of trading prices is reflected in the evolution of anticipated earnings.

Mathematically, FMH is equivalent to is the fact that the logprice $p_t = \ln P_t$ is a fractional Brownian motion with the following properties:

- $E(p_t) = 0, \forall t$.
- $E(p_t p_s) = \frac{1}{2}(t^{2H} + s^{2H} - |t-s|^{2H})$, $\forall t, s$.

In the above notations H is the Hurst coefficient, $H \in (0,1)$, which defines the behaviour of the returns series.

- *H=0.5* - prices follow a random walk process, and returns are not correlated;
- *H<0.5* - the series of returns presents positive autocorrelation (persistent series);
- *H>0.5* - the series of returns presents negative autocorrelation (anti-persistent series).

The fractional Brownian motion has the property of auto-similarity of a fractal, because in distribution terms, we have:

$p_{at} \sim |a|^{2H} p_t$.

Stable distributions have a remarquable property: they allow for skewness and heavy tails and more, any linear combination of stable independent variables is also stable. In other words, the shape of distribution is preserved under linear transformation.

In the literature there are several parameterizations of stable distributions. For this paper we have chosen the parameterisation S0, in Nolan (2003)'s variant.

Thus, a variable X follow a stable distribution $S(\alpha, \beta, \gamma, \delta; 0)$ if its characteristic function has the form:

$$\phi(t) = \begin{cases} \exp(-\gamma^{\alpha}|t|^{\alpha}[1 + \\ + i\beta\tan(\frac{\pi\alpha}{2})sign(t)(|\gamma t|^{1-\alpha} - 1)] + i\delta t, \alpha \neq 1 \\ \exp(-\gamma|t|[1 + i\beta t\frac{2}{\pi}sign(t)(\ln(\gamma|t|)] + \\ + i\delta t, \alpha = 1 \end{cases} .$$

In the above notation $\alpha \in (0.2]$ is the characteristic parameter (tail index), $\beta \in [-1,1]$ is the skewness parameter, $\gamma \in (0,\infty)$ is the scale parameter and $\delta \in \mathbf{R}$ is the location parameter.

The behavior of stable distributions is driven by the values of stability index α : small values are associated to higher probabilities in the tails of the distribution.

2. DATA ANALYSIS

The sample used in analysis consists in daily data for BET Index from Bucharest Stock Exchange (3357 observations, covering the interval 2007-2011).

Starting from the observed price p_t, we compute the logreturns as

$$r_t = \log p_t - \log p_{t-1} .$$

For time series of logreturns, we estimate the stability index α for stable distribution and the Hurst exponent, using a rolling window of 250 trading days.

The Hurst exponent was estimated using the R/S analysis, while the tail index α was estimated using the time series regression method (Kutrouvelis, 1980).

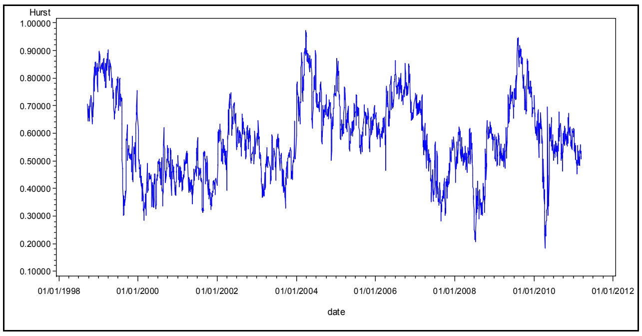

Figure 1. The Hurst exponent - 250 trading days rolling window

The behaviour of the Hurst exponent is not stable over time: starting 2008, when the financial crisis hit Romanian stock market, the time series returns present a very irregular pattern in terms of autocorrelation structure. The value of the Hurst exponent goes from 0.18 (positive autocorrelation) to 0.5 (random walk), having a maximum of 0.97 (negative autocorrelation).

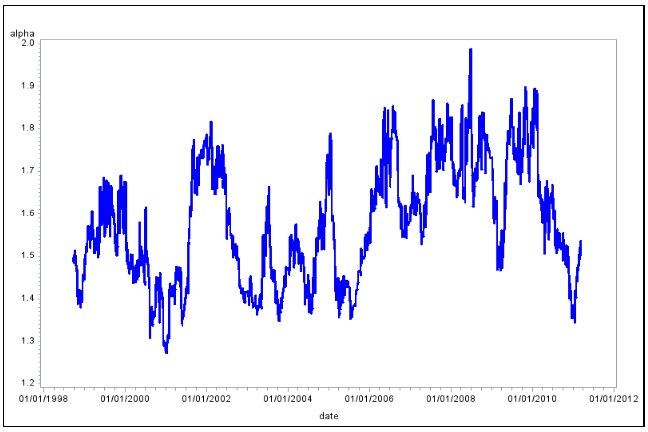

Figure 2. The tail index α - 250 trading days rolling window

The estimated values of the tail index for the returns of the BET Index show a clear departure from normality. The tail index has a maximum value of 1.98 on 30 June 2008, before the financial crisis in October 2008.

Our working hypothesis is that the likelihood of large negative returns could be explained by extreme values of the Hurst exponent or of the tail index. To verify this hypothesis we estimate the following binary logistic regression model:

$$P(Y_t^* = 1) = \frac{\exp(\beta_0 + \beta_1 dH_t + \beta_2 d\alpha_t)}{1 + \exp(\beta_0 + \beta_1 H_t + \beta_2 \alpha_t)}. \qquad (1)$$

In the above equation, we have:

- $Y_t^* = 1$, where $Y_t^* = \{1 \mid r_t < r_t^-, P(r_t < r_t^-) = 0.05\}$ (lower tail of returns distribution);

- $dH_t = H_t - 0.5$ is the deviation of the Hurst exponent at time *t* from the value corresponding to a random walk ;

- $d\alpha_t$ is the relative change of the tail index from the time *t-1* to *t*.

Table 3. Analysis of Maximum Likelihood Estimates for model (1)

Parameter	Estimate	Standard Error	Wald Chi-Square	Pr > ChiSq
Intercept	-2.86	0.09	1029.22	<.0001
dHurst	-1.50	0.58	6.70	0.01
dalpha	-12.78	6.14	4.34	0.04

Table 4. Odds Ratio Estimates for model (1)

Effect	Point Estimate	95% Wald Confidence Limits	
dHurst	0.22	0.07	0.69
dalpha	<0.001	<0.001	0.47

Analysing the results of the estimation, we can extract some facts regarding the relationship between the fractal nature of returns, heavy tailness and the probability of large negative returns:

- The likelihood of extreme negative returns decreases as the value of Hurst exponent is higher than 0.5, the characteristic value of a random walk. In other words, if the local trends on the market are likely to invert, i.e. the time series of returns is anti-persistent, then the probability of a stock market crash diminishes. By contrast, if the market has a persistent trend, then the likelihood of a stock market crash increases.
- If the market stability index increases (the tail index converges to Gaussianity), then the likelihood of a stock market crash decreases. By contrary, if the tail index has a negative trend, then the likelihood of a stock market crash increases.

In order to evaluate the impact of both fractal dimension and probability in the tails of the returns distribution, we set up an uncertainty index, as follows:

$$UI_t = \frac{H_t}{\alpha_t}.$$

For this uncertainty index we have the following situations:

- For a constant Hurst exponent, the Uncertainty Index is negatively correlated to departure from normality of returns distribution;
- For a constant tail index, high values of UI_t correspond to high values of Hurst exponent, meaning reversions in local trends.

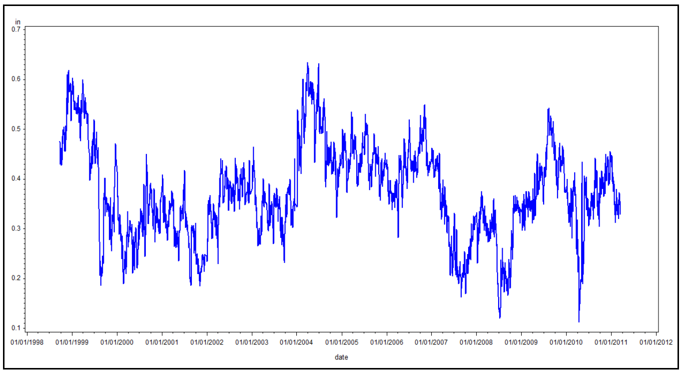

Figure 3. Uncertainty index for BET Index

The following binary logistic regression model

$$P(Y_t^* = 1) = \frac{\exp(\beta_0 + \beta_1 UI_t)}{1 + \exp(\beta_0 + \beta_1 UI_t)} \qquad (2)$$

was estimated and the results are presented in the following table.

Table 5. Analysis of Maximum Likelihood Estimates for model (2)

Parameter	Estimate	Standard Error	Wald Chi-Square	Pr > ChiSq
Intercept	-1.69	0.32	27.40	<.0001
Index	-3.54	0.90	15.35	<.0001

Table 6. Odds Ratio Estimates for model (2)

Effect	Point Estimate	95% Wald Confidence Limits	
Index	0.03	0.01	0.17

Analysing the results of the estimation, we can see that the Uncertainty Index is negatively correlated to the likelihood of extreme negative daily returns. Thus, if the Uncertainty Index increases by 1, then the odds of occurrence of extreme negative values of BET returns drops by around 97%.

3. CONCLUSIONS

In this paper we propose a method for defining a measure of stock market uncertainty, using the Hurst exponent and the tail index associated stable distributions. The statistical tests applied to the daily BET index returns series indicate that this measure of uncertainty is correlated to probability of large negative returns. In our future researches we intend to develop this uncertainty index in order to capture both fractal dimension and heavy tailness of returns distribution.

4. REFERENCES

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APPENDIX

A SAS code for estimating the Hurst exponent

This SAS code uses a dataset called *date*, with two variables: t is the time index and logreturn is the variable containing the returns of a financial asset.

The value of the Hurst exponent is saved in the dataset *hurst*.

```
proc iml;
use date;

READ all var {t logreturn} into y;
n=nrow(y);
p=int(log2(n/16));
pp=2**p;

rsp=j(pp,p,0);
nmare=j(p,1,0);

do q=0 to p;
      k=2**q;

      delta=int(n/k);
      xt=j(delta,1,0);
      st= j(delta,1,0);
      gt=j(delta,1,0);
      do i=1 to k;
            do j=1 to delta;

            xt[j]=y[(i-1)*delta+j,2];
            end;
              mean = mean(xt);

            std = sqrt(var(xt));


                  st=xt-mean;
                  gt[1]=st[1];
                  do l=2 to delta;
                  gt[l]=gt[l-1]+st[l];
      end;
                  gmax=max(gt);
                  gmin=min(gt);
                  rrs=(gmax-gmin)/std;
                        rsp[i,q+1]=rrs;
```

```
      nmare[q+1,1]=delta;

      end;

end;

nonzero=j(pp,p,0);


do i=1 to pp;
do j=1 to p;
if rsp[i,j]>0 then nonzero[i,j]=1;
end;
 end;

s=j(p,1,0);
nr=j(p,1,0);
rs=j(p,1,0);
do i=1 to p;
s[i]=sum(rsp[,i]);
nr[i]=sum(nonzero[,i]);
rs[i]=s[i]/nr[i];
end;
hurst=rs||nmare;
y=log(rs);
x1=log(nmare);

x0t=j(1,p,1);
/* x0t is a row vector of ones */
x=(x0t//x1`)`; /* xt is the transposed
design matrix for a simple linear
regression */
b=inv(x`*x)*x`*y; /* b is a vector of
estimated regression coefficients*/
hurst=b[2,1];
      create hurst from hurst;append from
hurst;
quit;
```