COMMON POINTS BETWEEN CREDIT RISK AND PHYSICS MODELLING

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Abstract. *Monte Carlo methods are a standard way to model both physics and credit risk. A particular type of credit risk, the contagion risk, is further explained as a contagion risk operator acting on credit risk states.*

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1. FINANCIAL DEFINITIONS

When lending cash/securities, one does lend a principal amount which will have to be completely returned, and will receive as well as additional cash or securities, usually named interest payments. “Credit risk refers to the risk that a borrower will default on any type of debt by failing to make payments which it is obligated to do. The risk is primarily that of the lender and includes lost principal and interest, disruption to cash flows, and increased collection costs. The loss may be complete or partial and can arise in a number of circumstances” [1].

The term *model* refers to a quantitative method, system, or approach that applies to statistical, economic, financial, or mathematical theories, techniques, and assumptions to process input data into quantitative estimates [2].

Except for credit risk, the other two most studied types of financial risk are the market and the liquidity risk. Market risk is defined as the risk faced due to changes in the market prices (ex: interest and exchange rates, stock and commodities prices). Liquidity risk is defined as the financial risk due to market liquidity (impossibility to sell the contract at the needed price) and due to funding liquidity (impossibility to get funded at the needed price). At present, there are two streams from credit risk, linking it to the market risk, via credit drivers, and to liquidity risk, via liquidity spreads, into an integrated market, credit and liquidity risk modelling.

The credit risk is usually measured by the CreditVar: the maximal loss faced due to credit risk events over a given time horizon and with a given probability. The faced loss is called exposure; it varies in time, and to get its present value one discounts its future value to the present time. Without entering into the details on what measure might be the best for the credit risk, we are going to focus on the causes and implications of particular type of credit risk: the contagion/systemic risk.

2. CREDIT RISK MODELLING

Up to a normalisation factor *N*, the credit risk CR_i , seen via the probability of default of counterparty *i,* is decomposed in a superposition of systematic risk, CRS_i, and idiosyncratic risk, CRI_i, at any moment *t:*

$$CR_i(t) = N * (CRS_i(t) \oplus CRI_i(t))$$

The systematic risk CRS_i depends on the credit risk drivers whose values are independent on the financial health of the counterparty *i*. These drivers are generally chosen among market prices (as commodity prices, indices, interest and exchange rates), among macroeconomic factors (as unemployment rate, gross domestic product per capita, inflation rate), or even the original credit rating state.

The idiosyncratic risk CRI_i is completely determined and depends entirely on the counterparty *i* and its financial health. It is normally modelled as white noise, via a scaled Gaussian distribution.

The standard approach in the literature [3] of specialty is credit risk modelling via factors model, where the first term gives rise to the systematic risk, and the second term gives rise to the idiosyncratic risk:

$$Y_i = \sum_{k=1}^{K} \beta_{ik} * Z_k + \sigma_i * \varepsilon_i$$

The variable Y, called credit worthiness index, is mapped, via a Gaussian copula, into the probability of default of counterparty *I* and becomes random by the incorporation of the standard Gaussian distribution ε_i. While this might be a mathematically attractive model, little can be said about the systematic factors *Z* and how the factor loadings β can be calculated in practice.

One can have a different description of the defaulting event if we look at it from the point of view of the hazard rate H_i. The probability of default at time t, $Q_i(t)$ is written as a function of the probability of survival, $S_i(t)$:

$$Q_i(t) = 1 - S_i(t)$$

Where:

$$S_i(t) = \exp\left(-\int_0^t H_i(\tau) \cdot d\tau\right) \qquad \text{(Eq. I)}$$

If we consider the exposure on the interval *(0,t]*, with respect to the counterparty *i* to be the sum between the number *v* of contracts of value *V*:

$$E_i(t) = \sum_{j=1}^{J} v_i^j * V_j(t)$$

We could write the credit risk at moment t as:

$$CR_i(t) = E_i * (1 - CRS_i(t) * CRI_i(t)) \qquad \text{(Eq. II)}$$

The systematic risk is the risk due to credit risk drivers:

$$CRS_i(t) = \exp\left(-\int_0^t \left(\sum_{k=1}^{K} a_G^k * X_k(\tau) \right) d\tau \right)$$

It is in practice hedged by derivatives or/and by netting agreements.

The idiosyncratic risk is meant to represent the diversification in the portfolio of counterparties:

$$CRI_i(t) = \exp\left(-\int_0^t (a_G^0 + \varepsilon_i) \cdot d\tau \right)$$

This represents the specific risk, completely related to the credited institution. A risk that is specific to a firm or industry can be theoretically canceled by diversification and it is called unsystematic or idiosyncratic risk.

In the equations above, X_k represent the K risk drivers to which counterparty i might be sensitive to, while the a^k are the regression factors when regressing the hazard rate H_i on the distributions of the risk drivers. The parameter a^0 represents the fact that counterparties from the group G, on which the regression is done, have a larger or smaller random, specific risk.

Once the regression factors are inferred from the existing data, knowing the forecasted behaviour of the risk drivers $X(t)$, the value and magnitude of the future probability of default and future credit risk can be Monte Carlo simulated and predicted.

3. SUBTYPES OF CREDIT RISK: SYSTEMIC / CONCENTRATION RISK

Systemic Risk is Contagion Risk. Systemic risk is generally used in reference to an event that can trigger a collapse in a certain industry or economy. It is related to the best studied type of credit risk: counterparty credit risk.

Even if liquidity risk is not considered to be a type of credit risk, there is a non-negligible correlation between liquidity and credit risk, because liquidity risk is mostly affected by and affects the systemic credit risk and the concentration risks. Here we will look at ways to understand the impact and formation of systemic risk. It is important to note that systemic risk is not due to individual institutions per se, but rather to the interconnections between the institutions. Systemic risk refers to the risks imposed by interlinkages and interdependencies in a system or market, where the failure of a single entity or cluster of entities can cause a cascading failure, which could potentially bankrupt or bring down the entire system or market [4, 5].

Systemic risks can arrise from market risk, credit risk dependency, funding concentration, exposures concentration and large specific/idyosynctratic risk.

In finance, systemic risk has been associated with a financial institution run which has a cascading effect on other financial institutions to which the first institutions owns money, causing a cascading failure. As depositors sense the ripple effects of default and liquidity concerns cascade through money markets, a panic can spread through a market, with a sudden flight to quality, creating many sellers but few buyers for the now illiquid assets. These interlinkages of the financial institutions runs are the issues which policy makers consider when addressing the issue of protecting a system against systemic risk.

The credit events experienced by the counterparties are the default and the changing in rating states. Because decreases in the rating state can be seen as a price variation and included in the market risk, we can simplify and limit the analysis of the systemic risk of the crediting financial institution at the risk strictly coming from the defaults of its counterparties. Unforeseen default can only take place when the idiosyncratic risks of different counterparties become correlated [6].

4. SYSTEMIC RISK EXEMPLIFIED

3.1 Interaction due to credit risk drivers

Mathematically, starting from Eq.II, concentration risk (correlated losses of counterparty i_1 and counterparty i_2) may show up whenever counterparties i_1 and i_2 are exposed to the same credit risk drivers X.

This leads to a systemic risk. And there is a causal dependency between any of the counterparties. If *A* is correlated to *A'* due to their common dependency on a given variable, say X=HMI (House Market Index). If the house market reaches a critical soil, both *A* and *A'* will default. But this is concentration risk and it takes into account when default is caused by common risk factors. When there is a non-causal correlation, even if *A* defaults, *A'* will not be affected.

The fact that counterparty *A* depends on counterparty *B*, who depends on counterparty *C*, who depends on counterparty *A* is called wrong way risk. It is the risk we are facing when having as counterparty one member of the cluster *ABC* (say lending the money to A) and hedge it by a deal opened with another member of the cluster (say gives as collateral a bond on C). In [7], one models the dependence relation between the default time of the counterparty and those of the underlying names in a reference portfolio and shows that there is an enormous joint impact of correlation and contagion due to the wrong way risk. We are not dealing hereby with wrong way risk, which is not systemic risk, even if it leads to it.

The only possibility that the systemic risk appears, in the absence of wrong way risk and concentration risk, is that all the idiosyncratic risks get affected at the same time.

3.2 Systemic Risk is Contagion Risk

Equally defined as the risk that financial difficulties at one or more banks spill over to a large number of other banks or the financial system as a whole, the risk of contagion is a different perception of systemic risk. Contagion Risk represents the financial institution's perspective of the risk for the bank to be affected by a systemic effect. Managing it firstly means identifying the factors that, affected by the contagion, will lead to the bank's losses due to contamination.

Sometimes merely called market risk, systematic risk is the risk inherent in the aggregate market that cannot be solved by diversification. Systematic risk refers to overall market risk.

The idiosyncratic risk due to the counterparty *i* is given by:

$$CRI_i(t) = \left[\exp(-a_G^0) * \exp(-\varepsilon_i)\right]^t$$

When the financial institutions FI_1 and FI_2 share the same counterparties:

$$CR(FI_1) = CR_1 = \sum_{i=1}^{I} CR_i(t)$$

$$= \sum_{i=1}^{I} E_i - \sum_{i=1}^{I} E_i * \exp(-a_G^0 t) * \left[\exp(-\varepsilon_i t)\right]$$

$$= C_1 - \sum_{i=1}^{I} C_2(a_{G1}^0; t) * \exp(-\varepsilon_i t)$$

$$= C_1^{1'} - \sum_{i=1}^{I} C_2^1(a_{G1}^0; t) * Y(\varepsilon_i; t)$$

$$CR(FI_2) = CR_2 = C_1^2 - \sum_{i=1}^{I} C_2^2(a_{G2}^0; t) * Y(\varepsilon_i; t)$$

In an ultraglobalised environment, the systemic risk arrises because the idyosincratic risk, given by the alleatory variable ε, stops being alleatory, as the financial institutions share the very same alleatory variables. Because this common idiosyncratic risk is due to diversification itself, via globalisation, it cannot be canceled by further diversification and has to be seen as a Systematic Risk, representing the contagion/systemic risk.

The survival probability with respect to a borrower/counterparty, as expressed in Eq.I can be rewritten as:

$$S=H*X^T$$

H could be interpreted as an operator on the state $X=(X_S, X_I)$, representing the financial institution in the frame of credit risk drivers. Here we have *K+1* sources of uncertainty: X_S is a vector of size *K*, X_I is a vector of size 1. Starting from the decomposition of H on the different credit drivers, one can fit their loading factors.

A first application of this decomposition is that, based on predicted values for the credit drivers, Monte Carlo simmulations can be employed in order to deduce the future values of credit risk faced by financial institutions.

As a second application, the idiosyncratic risks of two financial institutions can get correlated in absence of any common market risk factor when *the financial institutions have a lot of clients/counterparties in common.* Systemic risk is produced by globalisation as everybody is exposed to the whole world and to all the industries. Systemic risk is caused by and increases with increased idyosincratic risk. Because "Supervisors were not aware of the systemic implications of institutional funding and liquidity management, and how idiosyncratic risk (e.g., sub-prime credit risk) could quickly morph into a systematic liquidity risk for the financial system as a whole." [8]

5. CONCLUSIONS

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